

# 1 Thesis Title

Generalized Continuum Theories for modeling failure in materials.

## 2 Objectives

1. To propose a framework for nonlocal phase field damage model. The proposed framework includes
  - (a) Developing a phase field model for predicting damage in materials.
  - (b) Implementation of the proposed phase field damage model in the framework of nonlinear finite elements using a modified arc length method.
2. To perform a parametric study to illustrate the mesh convergence based on  $h$ -refinement and length scale parameter studies.
3. To do a comparative study of the proposed phase field damage model with the popular nonlocal gradient enhanced damage model.
4. To bring out the equivalence of proposed phase field damage model with existing phase field fracture model from literature and propose a new phase field fracture model.
5. To implement this new proposed phase field fracture model and validate with experimental results from literature.
6. To propose a finite strain extension of the gradient enhanced damage model.

## 3 Research Interest

Phase Field Method, Gradient Enhanced Damage Method, Damage and Fracture mechanics.

## 4 Journal Publications

- M. C. Nataraja, **Karthik S.** and A. N. Madhusudan. Characterisation of manufactured fine aggregates based on uncompacted void content as per ASTM C 1252-03 – Urgent need for Indian scenario. Indian Concrete Journal, 90(1), 59-68, 2016.
- M. C. Nataraja, **Karthik S.** and A. N. Madhusudan. Characterisation of crushed fine aggregate for use in concrete based on methylene blue test. Indian Concrete Journal, 92(5), 18-25, 2018.
- **S. Karthik**, A. Rajagopal and J. N. Reddy. Nonlocal phase field approach for modeling damage in brittle materials. Mechanics of Materials, 103797, 2021.  
<https://doi.org/10.1016/j.mechmat.2021.103797>

- **Karthik S.**, K. S. S. Reddy, A. Nasedkina, A. Nasedkin and A. Rajagopal. Phase Field vs Gradient Enhanced Damage Models: A Comparative Study. *Procedia Structural Integrity*, 35, 173-180, 2022.  
<https://doi.org/10.1016/j.prostr.2021.12.062>
- **S. Karthik**, A. Nasedkina, A. Nasedkin and A. Rajagopal. Framework and numerical algorithm for a phase field fracture model. *East Asian Journal on Applied Mathematics*, 13(1), 162-176, 2023.  
10.4208/eajam.280921.270722

## 5 Conference Presentations

- **S. Karthik**, A. Nasedkin, A. Nasedkina and A. Rajagopal. A phase field fracture model for quasi-brittle material. XXVIII scientific conference "modern information technologies: trends and prospects of development (SITO 2021)", 198-201, 2021.  
<https://www.elibrary.ru/item.asp?id=46572860>
- **Karthik S.** and A. Rajagopal. A nonlocal phase field approach for modeling damage. SES 2019, 13-15 October 2019, Washington University in St. Louis, USA.
- **Karthik S.**, K. S. S. Reddy, A. Nasedkina, A. Nasedkin and A. Rajagopal. Phase field vs Gradient enhanced damage models: A comparative study. IWPDF 2021, 18-20 August 2021, Middle East Technical University, Ankara, Turkey.
- K. S. S. Reddy, **Karthik S.**, A. Nasedkina, A. Nasedkin and A. Rajagopal. Study on dynamic crack propagation in brittle materials using phase-field model. IWPDF 2021, 18-20 August 2021, Middle East Technical University, Ankara, Turkey.
- **Karthik S.**, K. S. S. Reddy, A. Nasedkina, A. Nasedkin and A. Rajagopal. A Nonlocal Phase Field Model for Fracture. MECHCOMP7, 1-3 September 2021, Faculty of Engineering, University of Porto, Portugal.
- K. S. S. Reddy, **Karthik S.**, A. Nasedkina, A. Nasedkin and A. Rajagopal. Phase-field modeling of brittle fracture with inertia effects. MECHCOMP7, 1-3 September 2021, Faculty of Engineering, University of Porto, Portugal.

## 6 Poster Presentation

- **Karthik S.**, Phase field and Gradient Enhanced Damage models: A numerical comparative study, Workshop on Non-Classical Advanced Mechanics of Materials (NCAMM 2019), IISc Bangalore, 9-11 July, 2019.

## 7 Achievements in Academics

1. Awarded the "Outstanding Civil Engineer" by the Association of Consulting Civil Engineers (ACCE), Bangalore chapter for being the Topper in Civil Engineering course among all colleges in Bangalore coming under VTU Belgaum for the year 2013.

2. Awarded the “Best M.Tech Thesis” by the Association of Consulting Civil Engineers (ACCE), Mysore chapter for the year 2015.

## 8 Relevant photos, diagrams, flowcharts

### Modified Arc-Length solution algorithm.

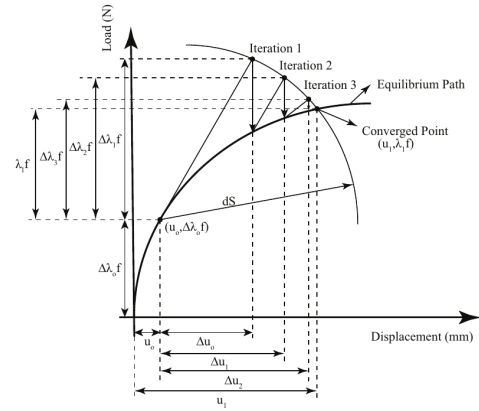
**Algorithm 1:** Modified Arc-Length Algorithm

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Result: Compute  $u_1$ 
Initialize  $\Delta u = 0$ , displacement  $u_0$ , load  $f$  and load increment factor  $\Delta\lambda_0 = 0.005$ 
if  $ITER=0$  then
  Compute internal force  $F_i^{u^*}$ ,  $F_i^\phi$ , residuals  $R_{u^*}$  and  $R_\phi$  and tangent stiffness  $K_{ij}^{u^*u^*}$ 
  and  $K_{ij}^{\phi\phi}$ .
  Compute corrector displacement factors  $\{du_r\} = [K_T]^{-1}\{R\}$  and
   $\{du_f\} = [K_T]^{-1}\{f\}$ 
  if  $STEP=0$  then
    Compute the Arc-length,  $dS = \Delta\lambda_0 \sqrt{\{du_f\}^T \{du_f\}}$ 
    Compute the incremental displacement  $\{\Delta u_0\} = \Delta\lambda_0 \{du_f\}$ 
    Update displacement  $\{u_1\} = \{u_0\} + \{\Delta u_0\}$ 
  else
    Take  $\Delta\lambda_1 = 0.005$ 
    Compute  $dS = \Delta\lambda_1 \sqrt{\{du_f\}^T \{du_f\}}$ 
    Compute  $\{\Delta u_1\} = \{du_r\} + \Delta\lambda_1 \{du_f\}$ 
    Update displacement  $\{u_1\} = \{u_0\} + \{\Delta u_1\}$ 
  end
else
  Update new incremental load factor  $\Delta\lambda_1 = \frac{(dS)^2 - (\Delta u_1)^T (\{\Delta u_0\} + \{du_r\})}{\{\Delta u_1\}^T \{du_f\}}$ 
  Compute  $\{\Delta u_1\} = \{du_r\} + \Delta\lambda_1 \{du_f\}$ 
  Update displacement  $\{u_1\} = \{u_0\} + \{\Delta u_1\}$ 
end

```

(a)



(b)

Figure 1: Modified Arc-length method: (a) Algorithm; (b) Graphical representation.

### Phase Field Damage Model

$$\frac{\partial \phi}{\partial t} = M \left[ -30\phi(\phi - 2\phi^2 + \phi^3) \frac{1}{2} \bar{\epsilon} : \bar{C} : \bar{\epsilon} + 2W\phi(1 + 2\phi^2 - 3\phi) - Wl^2 (\nabla^2 \phi) \right]$$

- Example: 1D bar,  $E=200\text{GPa}$ ,  $L=100\text{mm}$  and  $A=10\text{mm}^2$ .

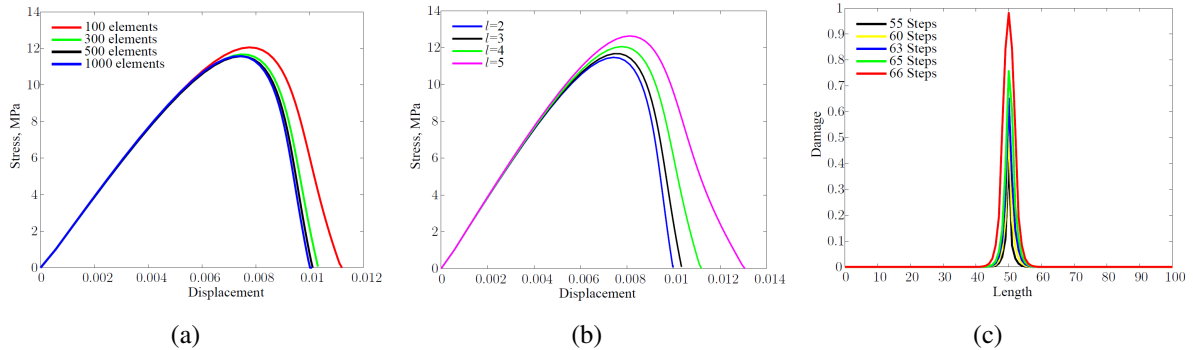


Figure 2: 1D bar example: (a) h-refinement studies; (b) length scale studies; (c) Damage profile

## Phase Field Fracture Model

$$\left[ \frac{2G_c}{\ell}(2\phi^2 - 3\phi + 1) + 30(2\phi^2 - \phi^3 - \phi)\psi_0^+ \right] \phi - G_c \ell \nabla^2 \phi = 0,$$

- Example: Symmetric double notch specimen,  $E = 210 \text{ kN/mm}^2$ ,  $\mu = 0.3$ ,  $G_c = 0.0027 \text{ kN/mm}$  and  $\ell = 0.0075 \text{ mm}$ .

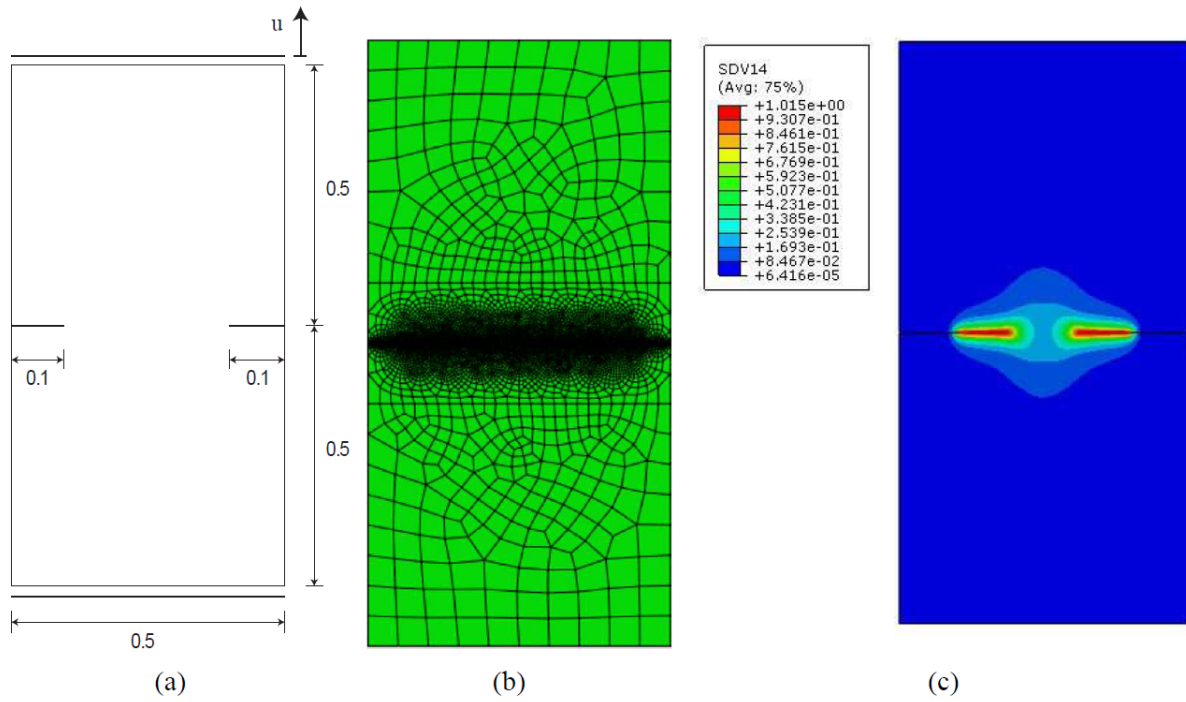


Figure 3: Double notch specimen: (a) Geometry; (b) Mesh; (c) Fracture pattern